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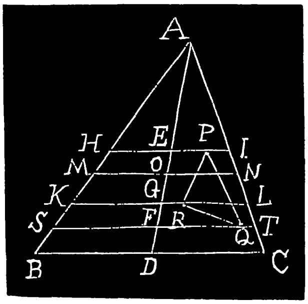
$$=\frac{16a^3}{9}\int_0^{\frac{1}{2}\pi}(3\theta\sin\theta\cos\theta-3\sin^2\theta+\sin^4\theta+10\sin^6\theta-8\sin^8\theta)d\theta,$$

$$=\frac{16a^3}{9}\left(\tfrac{3}{8}\pi-\tfrac{3}{4}\pi+\tfrac{3}{16}\pi+\tfrac{25}{16}\pi-\tfrac{35}{32}\pi\right)=\frac{16a^3}{9}\times\frac{9\pi}{32}=\tfrac{1}{2}\pi a^3.$$

117. Proposed by **G. B. M. ZERR, A. M., Ph. D.,** Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A straight line is drawn at random parallel to the base of a given triangle. Three random points are then taken, one on each side of the random line and one anywhere in the triangle. Find the average area of the triangle formed by the three random points.

Solution by the PROPOSER.



Let ABC be the given triangle, MN the random line parallel to the base BC. Take P above, Q below MN, and R anywhere. Through P, Q, R draw HI, ST, KL parallel, respectively, to MN. Let $BC=2a$, $BD=DC=a$, $AD=h$, $\angle ADC=\beta$, $AE=u$, $AF=v$, $AG=w$, $EP=x$, $GR=z$, $FQ=y$, $AO=r$.

Then $EI=\dfrac{au}{h}=x'$, $GL=\dfrac{aw}{h}=z'$, $FT=\dfrac{av}{h}=y'$, $GJ=y-\dfrac{(y-x)(v-w)}{v-u}=t$.

Area $PQR=\frac{1}{2}(t-z)(v-u)\sin\beta=A$, $t>z$; area $PQR=\frac{1}{2}(z-t)(v-u)\sin\beta=A_1$, $t<z$. The limits of r are 0 and h; of v, r and h; of u, 0 and r; of w, u and v; of x, $-x'$ and x'; of y, $-y'$ and y'; of z, $-z'$ and t, and t and z'.

$$\therefore\triangle=\frac{\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\left[\int_{-z'}^{t}Adz+\int_t^{z'}A_1dz\right]drdvdydudxdw}{\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\int_{-z'}^{z'}drdvdydudxdwdz}$$

$$=\frac{15}{a^3h^4}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\left[\int_{-z'}^{t}Adz+\int_t^{z'}A_1dz\right]drdvdydudxdw$$

$$=\frac{15\sin\beta}{2a^3h^6}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\int_u^v\left[h^2t^2+a^2w^2\right](v-u)drdvdydudxdw$$

$$=\frac{5\sin\beta}{2a^3h^6}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r\int_{-x'}^{x'}\left[a^2(u^2+uv+v^2)+h^2(x^2+xy+y^2)\right]$$

$$\times(v-u)^2drdvdydudx$$

$$=\frac{5\sin\beta}{3a^2h^7}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r(4a^2u^3+3a^2u^2v+3a^2uv^2+3h^2uy^2)(v-u)^2drdvdydu$$

$$=\frac{5\sin\beta}{36a^2h^7}\int_0^h\int_r^h\int_{-y'}^{y'}[3a^2r^2v^2-12a^2rv^3+18a^2v^4-12a^2r^3v+8a^2r^4$$

$$+3h^2y^2(6v^2-8rv+3r^2)]r^2drdvdy$$

$$=\frac{5a\sin\beta}{9h^8}\int_0^h\int_r^h(3r^4v^3-10r^3v^4+12r^2v^5-6r^5v^2+4r^6v)drdv$$

$$=\frac{5a\sin\beta}{36h^8}\int_0^h(3r^2h^4-8rh^5+8h^6-8r^3h^3+8r^4h^2-3r^6)r^2dr$$

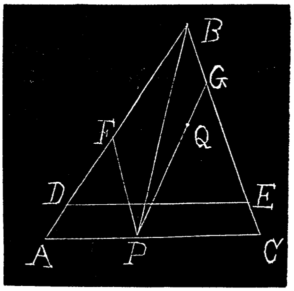
$$=\tfrac{13}{126}ah\sin\beta=\tfrac{13}{126}\text{ (area of triangle).}$$

118. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Find the mean distance between two points taken at random in an equilateral triangle.

I. Solution by L. C. WALKER. A. M., Graduate Student. Leland Stanford University. Cal.

Let ABC be any plane triangle whose sides opposite the angles A, B, C are respectively a, b, c. Let M be the required mean distance, and M_1 be the mean distance between the two points when one of the points is confined to one of the sides of the triangle, as AC. Let P be any point in the side AC, and let Q be any point in the surface of the triangle whose area$=\triangle$. Draw DE parallel to AC, and draw PF, PB, and PQG. Put $BD=u$, $AP=x$, $PQ=y$, $PF=z$, $PG=v$, $\angle BPC=\phi$, and $\angle FPC$ or $\angle GPC=\theta$. Then



$$v=\frac{c\sin A\sin(\phi+C)}{\sin\phi\sin(\theta+C)},\quad z=\frac{c\sin A\sin(\phi-A)}{\sin\phi\sin(\theta-A)},$$

$$x=\frac{c\sin(\phi-A)}{\sin\phi},\text{ and }dx=\frac{c\sin A}{\sin^2\phi}d\phi.$$

$$\text{Hence }M=\frac{\int_0^c M_1.\frac{u}{c}.\frac{u^2}{c^2}\triangle.DE.du}{\int_0^c\frac{u^2}{c^2}\triangle.DE.du}=\tfrac{4}{5}M_1$$

$$=\frac{4}{5b\triangle}\int_0^b\left[\int_0^\phi\int_0^v y^2dyd\theta+\int_\phi^\pi\int_0^z y^2dyd\theta\right]dx$$